<u>Natural Logic in Natural Language Inference</u>

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1. Building up to Natural Logic

Monotonicity is the starting point of natural logic. But Natural logic stems specifically from all the problems generated by only looking to monotonicity

a. Nobody can enter without a valid passport ⊨ Nobody can enter without a passport.
b. Whiskers is a cat ⊨ Whiskers is not a poodle.

But monotonicity does not give us anyway to handle this – it lacks <u>semantic exclusion.</u>

Sánchez Valencia (1991) :
entailment is <u>semantic containment</u> relation $\sqsubseteq$ analogous to the <u>set containment</u> relation $\subseteq$

2-way entailment:

$$\begin{aligned} \text{ENTAILMENT} &\stackrel{\text{def}}{=} \{\langle p, h\rangle \in \mathbf{Dom}_T^2 : p \models h\} \\ \text{NON-ENTAILMENT} &\stackrel{\text{def}}{=} \{\langle p, h\rangle \in \mathbf{Dom}_T^2 : p \not\models h\} \end{aligned}$$

3-way entailment

$$\begin{aligned} \text{ENTAILMENT} &\stackrel{\text{def}}{=} \{\langle p, h\rangle \in \mathbf{Dom}_T^2 : p \models h\} \\ \text{CONTRADICTION} &\stackrel{\text{def}}{=} \{\langle p, h\rangle \in \mathbf{Dom}_T^2 : p \models \neg h\} \\ \text{COMPATIBILITY} &\stackrel{\text{def}}{=} \{\langle p, h\rangle \in \mathbf{Dom}_T^2 : p \not\models h \wedge p \not\models \neg h\} \end{aligned}$$

Let's cut the cake, and see how these things differ:

<table>
<tr><th></th><th>2-way</th><th>3-way</th><th>containment</th></tr>
<tr><td>p. X is a couch
h. X is a sofa</td><td rowspan="2">ENTAILMENT</td><td rowspan="2">ENTAILMENT</td><td>p ≡ h</td></tr>
<tr><td>p. X is a crow
h. X is a bird</td><td>p ⊏ h</td></tr>
<tr><td>p. X is a fish
h. X is a carp</td><td rowspan="3">NON-ENTAILMENT</td><td rowspan="2">COMPATIBILITY</td><td>p ⊐ h</td></tr>
<tr><td>p. X is a hippo
h. X is hungry</td><td rowspan="2">NO-CONTAINMENT</td></tr>
<tr><td>p. X is a cat
h. X is a dog</td><td>CONTRADICTION</td></tr>
</table>

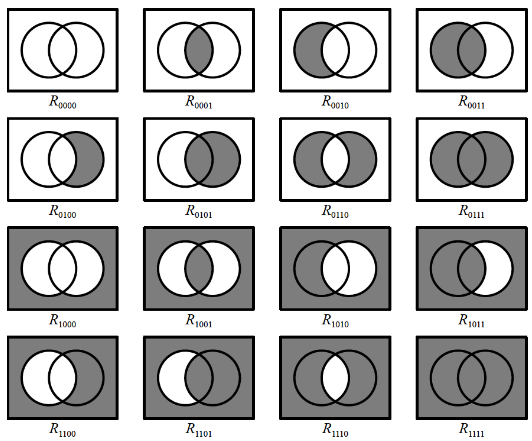
<u>Best of both worlds</u>:

Problem: A universe U contains $2^{|U|}$ sets, $4^{|U|}$ ordered pairs of sets, and thus $2^{4^{|U|}}$ possible set relations.

We want:
(a) include familiar and useful relations expressing equivalence, containment, and exclusion
(b) form a partition of the space of ordered pairs of sets (disjoint cover)

label	definition	meaning
00	$\overline{x} \cap \overline{y}$	in neither x nor y
01	$\overline{x} \cap y$	in y but not x
10	$x \cap \overline{y}$	in x but not y
11	$x \cap y$	in both x and y

For any two p and h: we realize that in a typical Venn diagram there are 4 regions,
We take 4 operations, and think about what they might be. We get $2^4 = 16$ cases, drawn below:



In math:

relation	constraint on x	constraint on y	constraint on $\langle x, y\rangle$
R_{0000}	$\emptyset = x = U$	$\emptyset = y = U$	$x = y$
R_{0001}	$\emptyset \subset x = U$	$\emptyset \subset y = U$	$x = y$
R_{0010}	$\emptyset \subset x = U$	$\emptyset = y \subset U$	$x \supset y$
R_{0011}	$\emptyset \subset x = U$	$\emptyset \subset y \subset U$	$x \supset y$
R_{0100}	$\emptyset = x \subset U$	$\emptyset \subset y = U$	$x \subset y$
R_{0101}	$\emptyset \subset x \subset U$	$\emptyset \subset y = U$	$x \subset y$
R_{0110}	$\emptyset \subset x \subset U$	$\emptyset \subset y \subset U$	$x \cap y = \emptyset \wedge x \cup y = U$
R_{0111}	$\emptyset \subset x \subset U$	$\emptyset \subset y \subset U$	$x \cap y \neq \emptyset \wedge x \cup y = U$
R_{1000}	$\emptyset = x \subset U$	$\emptyset = y \subset U$	$x = y$
R_{1001}	$\emptyset \subset x \subset U$	$\emptyset \subset y \subset U$	$x = y$
R_{1010}	$\emptyset \subset x \subset U$	$\emptyset = y \subset U$	$x \supset y$
R_{1011}	$\emptyset \subset x \subset U$	$\emptyset \subset y \subset U$	$x \supset y$
R_{1100}	$\emptyset = x \subset U$	$\emptyset \subset y \subset U$	$x \subset y$
R_{1101}	$\emptyset \subset x \subset U$	$\emptyset \subset y \subset U$	$x \subset y$
R_{1110}	$\emptyset \subset x \subset U$	$\emptyset \subset y \subset U$	$x \cap y = \emptyset \wedge x \cup y \neq U$
R_{1111}	$\emptyset \subset x \subset U$	$\emptyset \subset y \subset U$	$x \cap y \neq \emptyset \wedge x \cup y \neq U$
			$\ldots \wedge x \not\subseteq y \wedge x \not\supseteq y$

But, 16 is still kind of a lot.
Lucky for us, *Contradictions and tautologies may be common in logic textbooks, but they are rare in everyday speech.*

Examples:

R_{0000} is an extremely degenerate case: Universe is empty.

Singleton degeneracies:

Relations R_{0001}, R_{0010}, R_{0100}, and R_{1000} cover the cases and both x and y are either empty or universal.

i.e. X is a female or not. X is a man or not.

9 of these cases are "degenerate cases" due to properties of natural language.
The nine relations in R mentioned so far (namely, R_{0000}, R_{0001}, R_{0010}, R_{0011}, R_{0100}, R_{0101}, R_{1000}, R_{1010}, and R_{1100}) are boundary cases in which either x or y is either empty or universal.

So, we get rid of these 9. We are left with 7:

symbol[10]	name	example	set theoretic definition[11]	in $\mathfrak{R}$
$x \equiv y$	equivalence	*couch* $\equiv$ *sofa*	$x = y$	R_{1001}
$x \sqsubset y$	forward entailment	*crow* $\sqsubset$ *bird*	$x \subset y$	R_{1101}
$x \sqsupset y$	reverse entailment	*Asian* $\sqsupset$ *Thai*	$x \supset y$	R_{1011}
$x \wedge y$	negation	*able* $\wedge$ *unable*	$x \cap y = \emptyset \wedge x \cup y = U$	R_{0110}
$x \mid y$	alternation	*cat* $\mid$ *dog*	$x \cap y = \emptyset \wedge x \cup y \neq U$	R_{1110}
$x \smile y$	cover	*animal* $\smile$ *non-ape*	$x \cap y \neq \emptyset \wedge x \cup y = U$	R_{0111}
$x \# y$	independence	*hungry* $\#$ *hippo*	(all other cases)	R_{1111}

How to convert from Set relations to Entailment relations :
1- Restrict on types
2- x and y belong to relation R_{1101} iff y holds in every model where x holds (but not vice-versa)

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2. Compositional Semantics.

Joins

$$R \bowtie S \stackrel{\text{def}}{=} \{\langle x, z\rangle : \exists y \ (\langle x, y\rangle \in R \wedge \langle y, z\rangle \in S)\}$$

Some joins are clear:

$$\sqsubset \bowtie \sqsubset = \sqsubset$$
$$\sqsupset \bowtie \sqsupset = \sqsupset$$
$$\wedge \bowtie \wedge = \equiv$$
$$\forall R \quad R \bowtie \equiv = R$$
$$\forall R \quad \equiv \bowtie R = R$$

Not all joins are deterministic:

$x \mid y$	$y \mid z$	$x \; ? \; z$
$gasoline \mid water$	$water \mid petrol$	$gasoline \equiv petrol$
$pistol \mid knife$	$knife \mid gun$	$pistol \sqsubset gun$
$dog \mid cat$	$cat \mid terrier$	$dog \sqsupset terrier$
$rose \mid orchid$	$orchid \mid daisy$	$rose \mid daisy$
$woman \mid frog$	$frog \mid Eskimo$	$woman \# Eskimo$

So, for now, there is a method for computing joins – both deterministic and not.

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3. Compositional Semantics.

If two linguistic expressions differ by a single atomic edit (deletion, insertion, or substitution), then the entailment relation between them depends on two factors:

1. The lexical entailment relation generated by the edit;
$$\beta(x, e(x)) = \beta(e) = X$$
2. How this lexical entailment relation is affected by semantic composition with the remainder of the expression (the context). Projectivity
$$\beta(x, y) = Y$$
$$\beta(f(x), f(y)) = ?$$

Note: assumption: tense and aspect matter little in inference

3. A. Entailment Relations: Substitution, Deletion, Insertion

Basic example:

$x = red\ car$

$e = sub(car, convertible)$

Then $\beta(e) = \sqsupset$ (because convertible is a hyponym of car).

If $e = del(red)$, then $\beta(e) = \sqsubset$ (because red is an intersective modifier).

Substitutions of open-class terms

Synonyms : $\equiv$ relation (sofa $\equiv$couch, happy $\equiv$ glad, forbid $\equiv$ prohibit);

hyponym-hypernym pairs : $\sqsubset$ relation (crow $\sqsubset$ bird, frigid $\sqsubset$ cold, soar $\sqsubset$ rise);

antonyms : | relation (hot |cold, rise | fall, advocate | opponent).

Example:

a = unmarried man

b = bachelor

$$\beta(sub(a, b)) = \equiv$$

Mostly use WordNet for Synonomy, hyponymy, antonymy, etc.

Substitutions of closed-class terms

Generalized Quantifiers: “Some”, “All”,

i.e. "every" entails "some"
I have four children ⊏ I have two children
I have four children | I have two children

Generic deletions and insertions

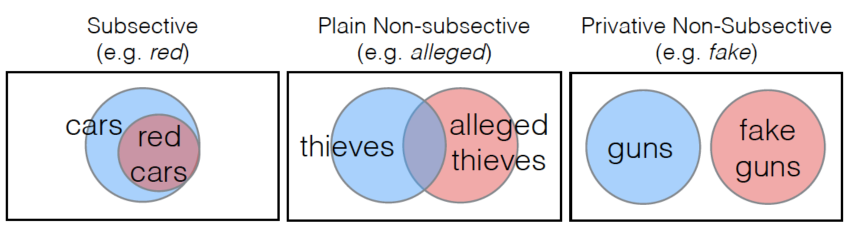
Generally governed by Monotonicity
Example: car which has been parked outside since last week ⊏ car

Special deletions and insertions

Factives vs implicatives
Implicatives:
Remember To: Two-way Implicative ++|--
Remember That: Factive
a. She remembered to lock the door. ENTAILS. She locked the door.
b. She did **not** remember to lock the door. ENTAILS. She did **not** lock the door.
c. She remembered that she locked the door. PRESUPPOSES. She locked the door.
d. She did **not** remember that she locked the door. PRESUPPOSES. She locked the door

Unfortunately – the process for determining factive vs implicative is not straightforward: "The sobering finding of this study that we are now in the progress of replicating with a more careful experimental design suggests that some very basic inferences such as whether the event described by an infinitival complement happened or not depend on opinions that are not part of the literal meaning of the sentence. This is a difficult problem for compositional semantics and for Natural Logic as well" – Kartunnen (2015)

Non-subsective adjectives
i.e. fake, former, and alleged. T
deleting fake or former seems to generate the | relation (fake diamond | diamond)
deleting alleged seems to generate the # relation (alleged spy # spy).



3. B. Semantic Composition : Lexical Edits: Projections

Projection - through Monotonicity: what we know already
Nobody can enter without pants – (nobody(can((without pants) enter)
Pants ⊏ Clothes
Without : ↓
Without↓ pants ⊐ Without clothes

Can : ↿

can↿ (without pants) enter ⊐ can (without clothes) enter

Nobody : ⇂

Nobody⇂ (can↿ (without ⇂ pants) enter) ⊏ Nobody(can(without clothes) enter)

Projection – generalized from monotonicity

$\beta(x, y) \in \{entailment\ relations\}$

$f \in \{connectives\}$

$\beta(f(x), f(y)) = ? \in \{entailment\ relations\}$

In theory, for each f there are 7^7 (823,543) possible entailment projections signatures:

$\equiv$	$\sqsubset$	$\sqsupset$	^	|	–	#
A	B	C	D	E	F	G

Let's look at projectivity of logical connectives:

connective	projectivity						
	$\equiv$	$\sqsubset$	$\sqsupset$	$\wedge$	$\mid$	$\smile$	#
negation (*not*)	$\equiv$	$\sqsupset$	$\sqsubset$	$\wedge$	$\smile$	$\mid$	#
conjunction (*and*) / intersection	$\equiv$	$\sqsubset$	$\sqsupset$	$\mid$	$\mid$	#	#
disjunction (*or*)	$\equiv$	$\sqsubset$	$\sqsupset$	$\smile$	#	$\smile$	#
conditional (*if*) (antecedent)	$\equiv$	$\sqsupset$	$\sqsubset$	#	#	#	#
conditional (*if*) (consequent)	$\equiv$	$\sqsubset$	$\sqsupset$	$\mid$	$\mid$	#	#
biconditional (*if and only if*)	$\equiv$	#	#	$\wedge$	#	#	#

Projectivity of quantifiers:

quantifier	projectivity for 1st argument							projectivity for 2nd argument						
	$\equiv$	$\sqsubset$	$\sqsupset$	$\wedge$	$\mid$	$\smile$	#	$\equiv$	$\sqsubset$	$\sqsupset$	$\wedge$	$\mid$	$\smile$	#
some	$\equiv$	$\sqsubset$	$\sqsupset$	$\smile^{\dagger}$	#	$\smile^{\dagger}$	#	$\equiv$	$\sqsubset$	$\sqsupset$	$\smile^{\dagger}$	#	$\smile^{\dagger}$	#
no	$\equiv$	$\sqsupset$	$\sqsubset$	$\mid^{\dagger}$	#	$\mid^{\dagger}$	#	$\equiv$	$\sqsupset$	$\sqsubset$	$\mid^{\dagger}$	#	$\mid^{\dagger}$	#
every	$\equiv$	$\sqsupset$	$\sqsubset$	$\mid^{\ddagger}$	#	$\mid^{\ddagger}$	#	$\equiv$	$\sqsubset$	$\sqsupset$	$\mid^{\dagger}$	$\mid^{\dagger}$	#	#
not every	$\equiv$	$\sqsubset$	$\sqsupset$	$\smile^{\ddagger}$	#	$\smile^{\ddagger}$	#	$\equiv$	$\sqsupset$	$\sqsubset$	$\smile^{\dagger}$	$\smile^{\dagger}$	#	#
at least two	$\equiv$	$\sqsubset$	$\sqsupset$	#	#	#	#	$\equiv$	$\sqsubset$	$\sqsupset$	#	#	#	#
most	$\equiv$	#	#	#	#	#	#	$\equiv$	$\sqsubset$	$\sqsupset$	$\mid$	$\mid$	#	#
exactly one	$\equiv$	#	#	#	#	#	#	$\equiv$	#	#	#	#	#	#
all but one	$\equiv$	#	#	#	#	#	#	$\equiv$	#	#	#	#	#	#

Example: most people were early | most people were late.

most fish talk # most birds talk

So… Notice a lot of #

Some **caveats** are in order.

Certain approximations have been made (except in the case of negation, which is exact).

<u>The projection of a given entailment relation can depend on the value of the other argument to the function</u>. That is, if we are given B(x; y), and we are trying to determine its projection B(f(x, z); f(y, z)), the answer can depend not only on the properties of f, but also on the properties of z.

x = French man y = European man

z= Parisian

4. Putting it all together : NatLog - ALGORITHM

1. Find a sequence of atomic edits <$e_1,\ldots,e_n$> which transforms p into h:
   h= $(e_n \circ \ldots \circ e_1) \circ p$ Let us say that $x_i = e_i \circ x_{i-1}$
2. For each e_i
   a. Determine the lexical entailment relation $\beta(e_i) = \beta(x_{i-1}, e_i(x_{i-1}))$
   b. Find the entailment relation $\beta(x_{i+1}, x_i)\ \forall\, i$
3. Join atomic entailment relations across the sequence of edits, as in section 5.6:
   $B(p,h) = B(x_0,x_n) = B(x_0,e_1) \bowtie \cdots \bowtie \mathrm{B}(x_{i-1},e_i) \bowtie \cdots \bowtie B(x_{n-1},e_n)$

System	P %	R %	Acc %
baseline: most common class	55.7	100.0	55.7
bag of words	59.7	87.2	57.4
NatLog 2007	68.9	60.8	59.6
NatLog 2008	89.3	65.7	**70.5**

Table 7.3: Performance of various systems on 183 single-premise FraCaS problems (three-way classification). The columns show precision and recall for the YES class, and accuracy.

References:

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